

Least Square fit

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Spring 2002

Linear Fit

> restart:

```
                                # standard Deviation
st_dev := sum( y[i - alpha0 - alpha1*x[i ] ]^2, i=1..N );
                                # minimum conditions
                                # first derivatives = 0

dstdev_d1 := diff( st_dev, alpha0
dstdev_d2 := diff( st_dev, alpha1

                                # solve equations for
minimum
eq1 := dstdev_d1 = 0;
eq2 := dstdev_d2 = 0;
solve( {eq1,eq2}, {alpha0,alpha1} );
```

$$st_dev := N\alpha_0^2 + \left(\sum_{i=1}^N (y_i^2 - 2y_i\alpha_0 - 2y_i\alpha_1 x_i + 2\alpha_0\alpha_1 x_i + \alpha_1^2 x_i^2) \right)$$

$$eq1 := 2N\alpha_0 + \left(\sum_{i=1}^N (-2y_i + 2\alpha_1 x_i) \right) = 0$$

$$eq2 := \sum_{i=1}^N (-2x_i y_i + 2\alpha_0 x_i + 2\alpha_1 x_i^2) = 0$$

$$\left\{ \alpha_1 = \frac{N \left(\sum_{i=1}^N x_i y_i \right) - \left(\sum_{i=1}^N x_i \right) \left(\sum_{i=1}^N y_i \right)}{N \left(\sum_{i=1}^N x_i^2 \right) - \left(\sum_{i=1}^N x_i \right)^2}, \alpha_0 = - \frac{- \left(\sum_{i=1}^N y_i \right) \left(\sum_{i=1}^N x_i^2 \right) + \left(\sum_{i=1}^N x_i \right) \left(\sum_{i=1}^N x_i y_i \right)}{N \left(\sum_{i=1}^N x_i^2 \right) - \left(\sum_{i=1}^N x_i \right)^2} \right\}$$

Quadratic fit

```
restart:
```

```
                                # standard Deviation
st_dev := sum( y[i] - alpha0 - alpha1*x[i] - alpha2*x[i]^2 )^2,
i=1..N );
```

```
# minimum conditions
```

```
# first derivatives = 0
```

```
dstdev_d1 := diff( st_dev, alpha0 );
```

```
dstdev_d2 := diff( st_dev, alpha1 );
```

```
dstdev_d3 := diff( st_dev, alpha2 );
```

```
# solve equations for
```

```
minimum
```

```
eq1 := dstdev_d1 = 0;
```

```
eq2 := dstdev_d2 = 0;
```

```
eq3 := dstdev_d3 = 0;
```

```
soln := solve( {eq1,eq2,eq3},{alpha0,alpha1,alpha2} );
```

```
# display solutions
```

```
assign( soln );
```

```
`Numerators of the three alpha coefficients`;
```

```
'alpha0';
```

```
numer(alpha0);
```

```
'alpha1';
```

```
numer(alpha1);
```

```
'alpha2';
```

```
numer(alpha2);
```

```
`denominator of the alpha coefficients (same for all three)`
```

```
denom(alpha1);
```

$$st_dev := N\alpha_0^2 + \left(\sum_{i=1}^N (y_i^2 - 2y_i\alpha_0 - 2y_i\alpha_1x_i - 2y_i\alpha_2x_i^2 + 2\alpha_0\alpha_1x_i + 2\alpha_0\alpha_2x_i^2 + \alpha_1^2x_i^2 + 2\alpha_1x_i^3\alpha_2 + \alpha_2^2x_i^4) \right)$$

$$eq1 := 2N\alpha_0 + \sum_{i=1}^N (-2y_i + 2\alpha_1x_i + 2\alpha_2x_i^2) = 0$$

$$\text{eq } \sum_{i=1}^N -2 \quad \alpha^2$$

Numerat the the lpha coeffic nt:
 α^0

$$\sum_{i=1}^N y_i \sum_{i=1}^N \left(\sum_{i=1}^N \right) \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N$$

$$\sum_{i=1}^N \left(\sum_{i=1}^N \left(\sum_{i=1}^N y_i \sum_{i=1}^N \sum_{i=1}^N y_i x_i \sum_{i=1}^N x_i \sum_{i=1}^N \sum_{i=1}^N \right) \right)$$

$$\sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N y_i \sum_{i=1}^N \sum_{i=1}^N \left(\sum_{i=1}^N \sum_{i=1}^N y_i \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N y \right)$$

α^2

$$\sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N$$

denominat of the alpha coeffic (same for al three)

$$\sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N \sum_{i=1}^N$$

$$\sum_{i=1}^N \sum_{i=1}^N$$